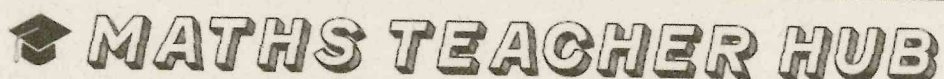


Name

ANSWERS

Class



www.MathsTeacherHub.com

Volume and surface area

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out**.
- If the question is a 1H question you are not allowed to use a calculator.
- If the question is a 2H or a 3H question, you may use a calculator to help you answer.

Information

- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question **carefully** before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

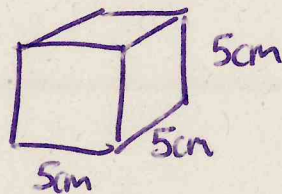
Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

- 3 A cube has a total surface area of 150 cm^2

Work out the volume of the cube.

$$150 \div 6 = 25 \text{ cm}^2$$

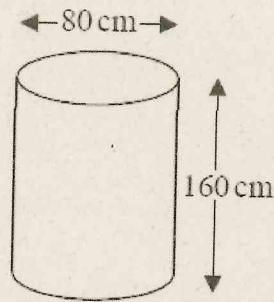
$$\sqrt{25} = 5 \text{ cm}$$



$$\begin{aligned} \text{Volume} &= 5 \times 5 \times 5 \\ &= 125 \end{aligned}$$

125 cm^3

- 4 Karina has 4 tanks on her tractor.
Each tank is a cylinder with diameter 80 cm and height 160 cm.



The 4 tanks are to be filled completely with a mixture of fertiliser and water.

The fertiliser has to be mixed with water in the ratio 1 : 100 by volume.
Karina has 32 litres of fertiliser.

101 parts

1 litre = 1000 cm³

Has Karina enough fertiliser for the 4 tanks?
You must show how you get your answer.

$$\text{Volume} = (\pi \times 40^2) \times 160 = 256000\pi$$

$$4 \text{ tanks} = 3216990.877 \text{ cm}^3$$

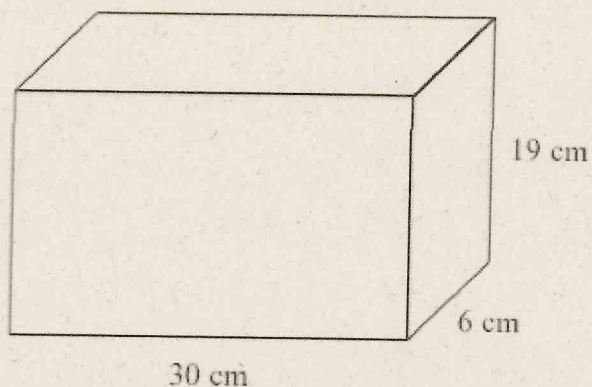
$$\frac{3216990.877}{1000} = 3216.99 \text{ litres}$$

$$\frac{3216.99}{101} = 31.85 \text{ litres}$$

Fertilizer : water
31.85 : 318.5

Yes Karina has enough to fill the 4 tanks

- 4 A container is in the shape of a cuboid.



The container is $\frac{2}{3}$ full of water.

A cup holds 275 ml of water.

What is the greatest number of cups that can be completely filled with water from the container?

$$\begin{aligned}\text{Volume} &= (30 \times 6) \times 19 = 3420 \text{ cm}^3 \\ &= 3420 \text{ ml}\end{aligned}$$

$$\frac{2}{3} \text{ of } 3420 = 2280 \text{ ml}$$

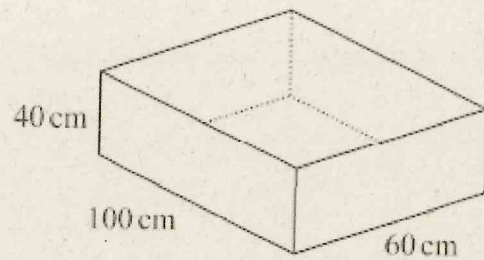
$$\frac{2280}{275} = 8.290$$

$$= 8 \text{ cups Filled}$$

8

- 5 The diagram shows a sand pit.
The sand pit is in the shape of a cuboid.

Sally wants to fill the sand pit with sand.
A bag of sand costs £2.50
There are 8 litres of sand in each bag.



Sally says,

"The sand will cost less than £70"

Show that Sally is wrong.

$$\begin{aligned}\text{Volume} &= (40 \times 100) \times 60 = 240000 \text{ cm}^3 \\ &= 240 \text{ litres}\end{aligned}$$

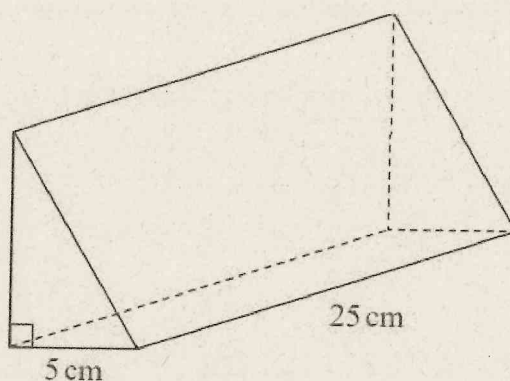
$$\frac{240}{8} = 30 \text{ bags of sand needed}$$

$$30 \times £2.50 = £75$$

Sally is wrong,

$$75 > 70$$

6 The diagram shows a prism.



The cross section of the prism is a right-angled triangle.
The base of the triangle has length 5 cm

The prism has length 25 cm
The prism has volume 750 cm^3

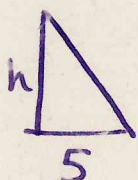
Work out the height of the prism.

$$\text{Volume} = \text{Area of cross section} \times \text{depth}$$

$$750 = \triangle \times 25$$

$$\frac{750}{25} = \triangle$$

$$30 = \triangle$$



$$\frac{5 \times h}{2} = 30$$

$$5 \times h = 60$$

$$h = 12$$

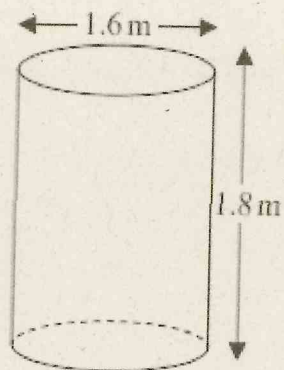
12 cm

6 Jeremy has to cover 3 tanks completely with paint.

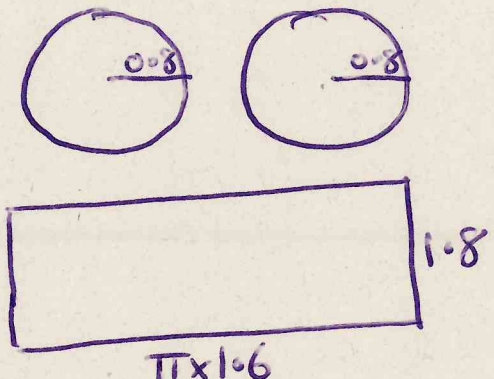
Each tank is in the shape of a cylinder with a top and a bottom.
The tank has a diameter of 1.6m and a height of 1.8m.

Jeremy has 7 tins of paint.
Each tin of paint covers 5m^2

Has Jeremy got enough paint to cover completely the 3 tanks?
You must show how you get your answer.



Surface area


$$\frac{16}{25}\pi + \frac{16}{25}\pi + \frac{72}{25}\pi = \frac{104}{25}\pi$$

$$\text{Total surface area} = 13.06902544\text{m}^2$$

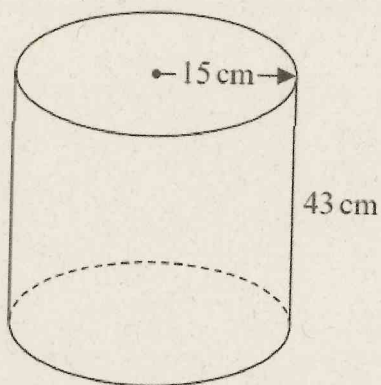
$$3 \text{ tanks} = 39.20707632\text{m}^2$$

$$7 \text{ tins} \times 5\text{m}^2 = 35\text{m}^2$$

Jeremy does not have enough to
to paint the tanks

$$39.2 > 35$$

- 7 The diagram shows an empty tank in the shape of a cylinder.



The cylinder has radius 15 cm and height 43 cm.

Water flows into the tank at a rate of 0.47 litres per minute.

Calculate the number of minutes it will take to completely fill the tank.
Give your answer correct to the nearest minute.

$$\text{Volume} = (\pi \times 15^2) \times 43 = 30395 \text{ cm}^3$$

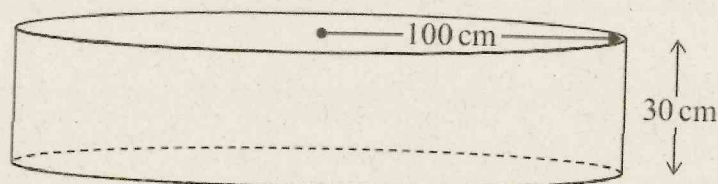
$$1000 \text{ cm}^3 = 1 \text{ litre}$$

$$30395 \text{ litres}$$

$$\frac{30395}{0.47} = 65 \text{ minutes}$$

65 minutes

- 7 A paddling pool is in the shape of a cylinder.



The pool has radius 100 cm.

The pool has depth 30 cm.

The pool is empty.

It is then filled with water at a rate of 250 cm^3 per second.

Work out the number of minutes it takes to fill the pool completely.

Give your answer correct to the nearest minute.

You must show all your working.

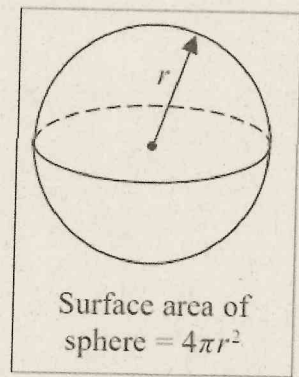
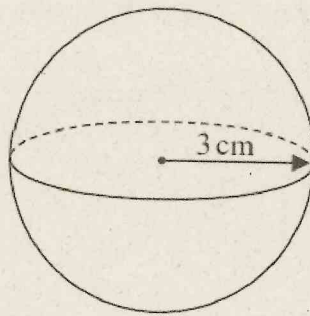
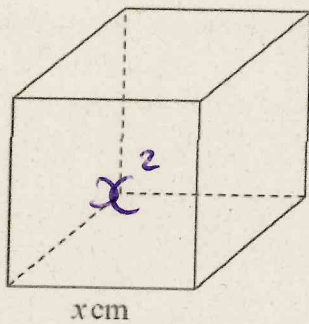
$$\text{Volume} = (\pi \times 100^2) \times 30 = 300000\pi \text{ cm}^3$$

$$\frac{300000\pi}{250} = 3770 \text{ seconds}$$

$$\frac{3770}{60} = 62.8 \text{ minutes}$$

63 minutes

- 7 The diagram shows a cube with edges of length x cm and a sphere of radius 3 cm.



The surface area of the cube is equal to the surface area of the sphere.

Show that $x = \sqrt{k\pi}$ where k is an integer.

$$\text{Surface area of cube} = 6x^2$$

$$\text{Surface area of sphere} = 4 \times \pi \times 3^2 = 36\pi$$

$$6x^2 = 36\pi$$

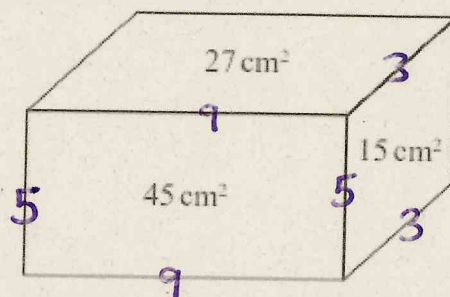
$$x^2 = 6\pi$$

$$x = \sqrt{6\pi}$$

$$k = 6$$

- 8 The diagram shows a solid metal cuboid.

The areas of three of the faces are marked on the diagram.
The lengths, in cm, of the edges of the cuboid are whole numbers.



$$\begin{aligned} 27 &= 3 \times 9 \\ 15 &= 3 \times 5 \\ 45 &= 5 \times 9 \end{aligned}$$

The metal cuboid is melted and made into cubes.
Each of the cubes has sides of length 2.5 cm.

Work out the greatest number of these cubes that can be made.

$$\begin{aligned} \text{Volume} &= (9 \times 5) \times 3 = 135 \text{ cm}^3 \\ \text{Cuboid} \end{aligned}$$

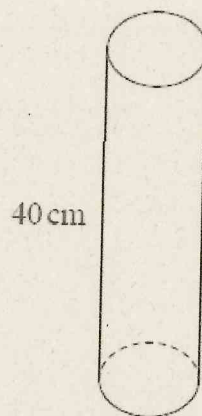
$$\begin{aligned} \text{Volume} &= (2.5 \times 2.5) \times 2.5 = 15.625 \text{ cm}^3 \\ \text{Cube} \end{aligned}$$

$$\frac{135}{15.625} = 8.64$$

8 cubes

8

- 8 The diagram shows a solid cylinder on a horizontal floor.



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The cylinder has a

volume of 1200 cm^3
height of 40 cm.

The cylinder exerts a force of 90 newtons on the floor.

Work out the pressure on the floor, due to the cylinder.

$$\text{Volume} = \text{Area} \times \text{depth}$$

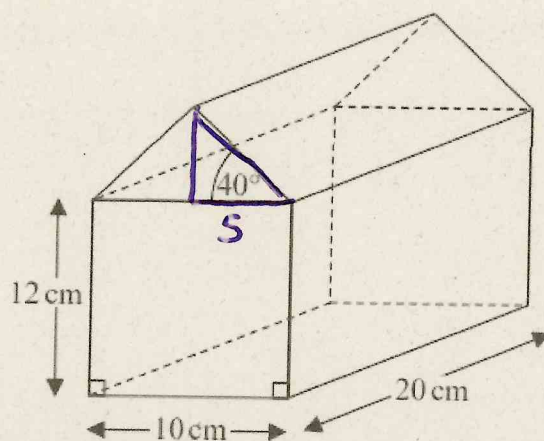
$$1200 = \text{Area} \times 40$$

$$30 = \text{Area}$$

$$\text{Pressure} = \frac{F}{A} = \frac{90 \text{ newtons}}{30 \text{ cm}^2}$$
$$= 3$$

$$3 \text{ newtons/cm}^2$$

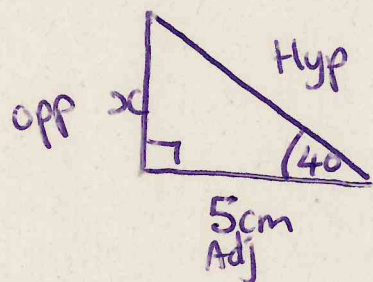
9 The diagram shows a prism.



The cross section of the prism has exactly one line of symmetry.

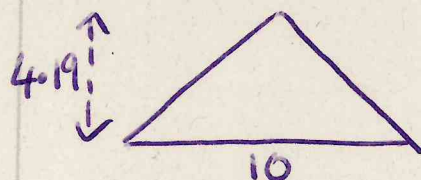
Work out the volume of the prism.

Give your answer correct to 3 significant figures.

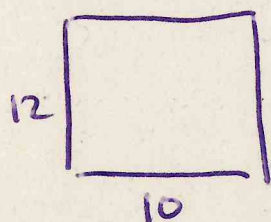


$$\tan(40) \times 5 = x$$

$$4.19549 = x$$



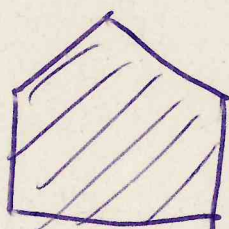
$$\frac{10 \times 4.19549}{2} = 20.97749 \text{ cm}^2$$



$$10 \times 12 = 120 \text{ cm}^2$$

$$V = 140.97749 \times 20$$

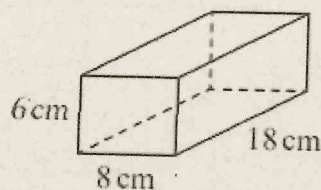
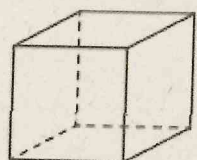
$$= 2819.5498 \text{ cm}^3$$



$$= 140.97749 \text{ cm}^3$$

$$2820 \text{ cm}^3$$

- 9 The diagram shows a cube and a cuboid.



The total surface area of the cube is equal to the total surface area of the cuboid.

Janet says,

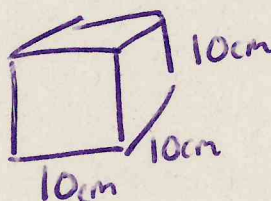
"The volume of the cube is equal to the volume of the cuboid."

Is Janet correct?

You must show how you get your answer.

$$\begin{aligned} \text{Cuboid Surface Area} &= (6 \times 8) + (8 \times 18) + (6 \times 18) \\ &= 48 + 144 + 108 = 300 \\ &\quad \times 2 = \underline{600 \text{ cm}^2} \end{aligned}$$

$$\begin{aligned} \text{Cube Surface Area} &= \frac{600}{6} = 100 \end{aligned}$$

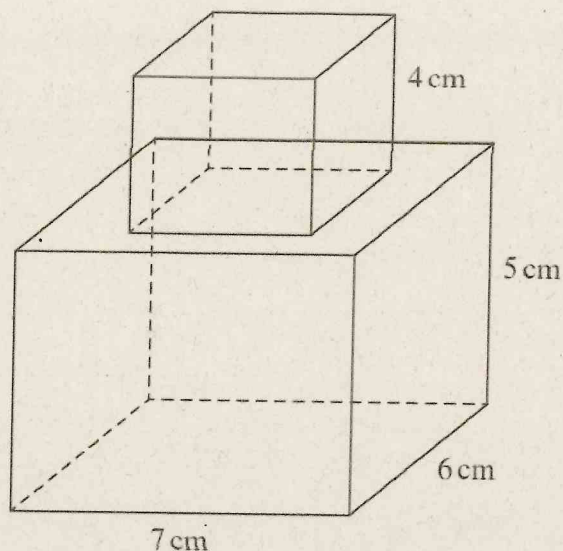


$$\begin{aligned} \text{Volume Cuboid} &= (6 \times 18) \times 8 \\ &= \underline{864 \text{ cm}^3} \end{aligned}$$

$$\begin{aligned} \text{Volume Cube} &= (10 \times 10) \times 10 \\ &= \underline{1000 \text{ cm}^3} \end{aligned}$$

Janet is wrong, they have different volumes.

- 9 A cube is placed on top of a cuboid, as shown in the diagram, to form a solid.



The cube has edges of length 4 cm.

The cuboid has dimensions 7 cm by 6 cm by 5 cm.

Work out the total surface area of the solid.

$$\text{Front} = 7 \times 5 = 35 \text{ cm}^2$$

$$\text{Back} = 7 \times 5 = 35 \text{ cm}^2$$

$$\text{Right} = 6 \times 5 = 30 \text{ cm}^2$$

$$\text{left} = 6 \times 5 = 30 \text{ cm}^2$$

$$\text{Base} = 7 \times 6 = 42 \text{ cm}^2$$

$$\text{Top} = (7 \times 6) - (4 \times 4) = 26 \text{ cm}^2$$

$$\text{Cube} = 4 \times 4 = 16$$

$$5 \text{ faces of } 16$$

$$= 80 \text{ cm}^2$$

$$\text{Total surface area} = 198 \text{ cm}^2 + 80 \text{ cm}^2$$

$$= 278 \text{ cm}^2$$

278

cm²

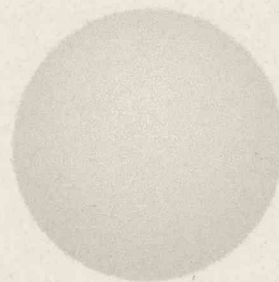
12 Here are three spheres.



P



Q



R

The volume of sphere Q is 50% more than the volume of sphere P.

The volume of sphere R is 50% more than the volume of sphere Q.

Find the volume of sphere P as a fraction of the volume of sphere R.

$$P = 100\text{cm}^3$$

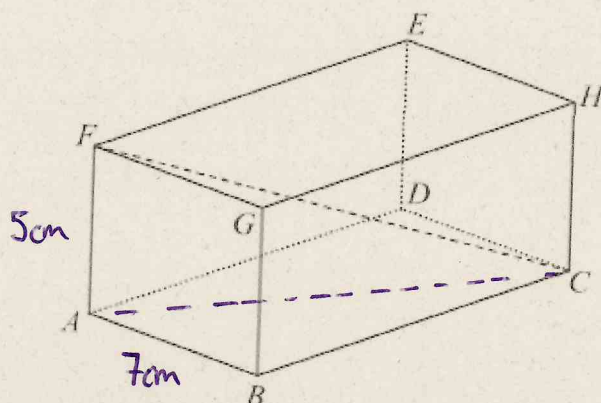
$$Q = 100 \times 1.5 = 150\text{cm}^3$$

$$R = 150 \times 1.5 = 225\text{cm}^3$$

$$\frac{P}{Q} = \frac{100}{225} = \frac{4}{9}$$

$$\frac{4}{9}$$

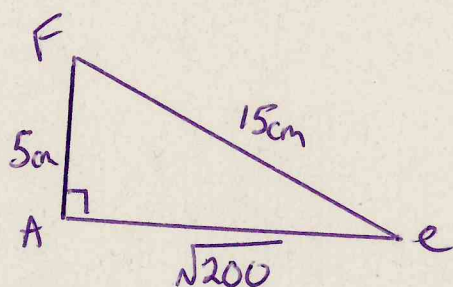
12 The diagram shows a cuboid $ABCDEFGH$.



$AB = 7$ cm, $AF = 5$ cm and $FC = 15$ cm.

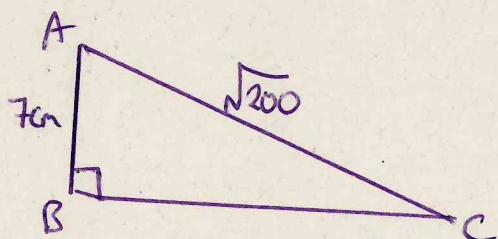
Calculate the volume of the cuboid.

Give your answer correct to 3 significant figures.



$$15^2 - 5^2 = 200$$

$$AC = \sqrt{200}$$



$$(\sqrt{200})^2 - 7^2 = 151$$

$$BC = \sqrt{151}$$

$$\text{Volume} = 35\text{cm}^2 \times \sqrt{151}$$

$$= 430.0872005$$

430

cm³

- 21 A solid cuboid has a volume of 300 cm^3
The cuboid has a total surface area of 370 cm^2

The length of the cuboid is 20 cm .

The width of the cuboid is greater than the height of the cuboid.

Work out the height of the cuboid.

You must show all your working.

$$l \times w \times h = 300$$

$$20 \times w \times h = 300$$

$$20wh = 300$$

$$wh = 15$$

$$\boxed{h = \frac{15}{w}}$$

or

$$\boxed{w = \frac{15}{h}}$$

Surface Area

$$2lw + 2lh + 2wh = 370$$

$$40w + 40h + 2wh = 370$$

$$40\left(\frac{15}{h}\right) + 40h + 2\left(\frac{15}{h}\right)h = 370$$

$$\frac{600}{h} + 40h + 30 = 370$$

$$\frac{600}{h} + 40h = 340$$

$$\frac{600}{h} + 40h - 340 = 0$$

$$600 + 40h^2 - 340h = 0$$

$$4h^2 - 34h + 60 = 0$$

$$2h^2 - 17h + 30 = 0$$

$$(2h-5)(h-6) = 0$$

$$h = 6$$

$$h = 2.5$$

height is smaller
than its width

2.5

cm

(Total for Question 21 is 5 marks)